

Spatial Reasoning Experiences in a STEM Context

8 Year-Old Disadvantaged Children Exploring Earthquakes and Related Phenomena

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Abstract

We present parts of a study that focused on the design and implementation of a learning trajectory in a STEM context for promoting spatial reasoning in 8 year-old disadvantaged children. The larger context of the learning trajectory was the earthquake phenomenon. It included tasks in which students built some concrete instruments (e.g., a seismograph) and structures, as well as used some digital tools (e.g., Google Maps, LightBot), in order to explore aspects related to the phenomenon (such as designing escape routes). By having authentic experiences and engaging in hands-on constructions, students were able to perceive or imagine and articulate different spatial sizes, experiment with different measurement strategies and use adequate measuring instruments for each spatial size.

Keywords and Phrases: Spatial reasoning, Space sizes, Reference frames, Primary school, Earthquake risk, Escape routes, Concrete materials, Google Maps, Google Earth, LightBot.

1. Introduction

This paper presents parts a STEM-based learning trajectory aimed to promote the development of spatial reasoning in primary-school children, in the context of earthquakes. This is a familiar context for students in our country, which is earthquake-prone. The study of this natural phenomenon involves spatial reasoning (Kastens & Ishikawa, 2006) and is rich for studying representations, spatial pattern recognition and awareness of different spaces. For example, constructionist tasks that involve designing or implementing escape plans require the interpretation and production of representations of different sizes of spaces (e.g., a classroom, a town) as well as reading maps and their symbology.

Spatial reasoning is part of everyday life (Harris, 2023), necessary for various professional fields and influences the cognitive development of students (Clements & Sarama, 2011). Spatial reasoning develops from an early age; in particular when a child observes, recognises and establishes proximity relationships between objects, places



and the spaces that contain them. It is essential that children become aware of these relationships and learn to communicate them (Freudenthal, 2002). The location of an object is relative to the position of the observer, and thus its description will vary (Freudenthal, 2002). That is, the subject needs to take a perspective by putting into action a reference frame –the system of relationships between objects, places and spaces. As described in the next section, reference frames are of three types: *egocentric*, *allocentric* and *decentred* (Tversky & Hard, 2009), and happen in various spaces – microspaces, mesospaces or macrospaces (Gálvez, 1985).

Davis et al. (2015) point out that school mathematics should aim to develop spatial reasoning skills, and that research is needed in terms of what type of tasks can promote that development in primary school. Thus, our research question was on *how to design tasks and learning contexts in primary school that foster students to develop and/or use spatial reasoning skills?* To do so, we designed the aforementioned learning trajectory in order to provide children with meaningful constructionist (Papert, 1991) experiences through a variety of hands-on tasks, using both concrete and digital resources, in the STEM context of earthquakes.

2. Conceptual Framework

Our conceptual framework takes into account two main aspects: (i) spatial reasoning skills and actions; and (ii) the sizes of spaces. With respect to the first, Davis et al. (2015) point to the need to simultaneously study both cognitive acts and physical / embodied actions. Those authors propose a system – represented by a wheeled diagram– to illustrate the complexity involved in spatial reasoning skills, that integrates “mental” understandings and “physical” transformations: *interpreting*, *(de)constructing*, *sensating*, *altering*, *moving* and *situating*. In particular, they recommend analysing how students develop interpretation skills when they engage in tasks that require moving objects or their own body in a given space to modify its position and orientation. Spaces can be of various sizes (micro, meso and macro spaces): Gálvez (1985) explains that in a microspace one can have a direct perception and manipulation of objects; in a mesospace (e.g., a house), the observation of fixed objects takes place from different reference points without the possibility of manipulating them; finally, in a macrospace (such as a city or a rural area) perception is neither global nor simultaneous, but through successive local visions.

Our learning trajectory seeks to involve children in activities that require engaging, constructing and moving in space: e.g., measuring lengths in micro- and mesospaces to estimate lengths in meso- and macrospaces, and interpreting and constructing representations. These types of activities involve *sensating* actions (e.g., *tactilizing*, *imagining*, *visualizing*, and *perspective-taking*) and *situating* actions (e.g., *dimensional shifting*, *locating* and *orientating*) (Davis et al., 2015). For example, the cognitive actions of estimating and iterating measurements can be fostered through physical experiences, in micro and mesospaces. On the other hand, perspective-taking involves recognizing the position and location of objects to understand the parts in relation to the whole. This process requires spatial structuring that involves identifying components, combining them and relating them to each other (Battista and

Clements, 1996). Furthermore, Tversky and Hard (2009) consider three reference systems through which objects and subjects can be perceived and represented from different perspectives: (i) the *egocentric* system, which depends on the observer (it is centred on the observer him/herself); (ii) *allocentric* (centred on the environment surrounding the object/subject); and (iii) *decentred* (centred on the perspective of another person).

3. Methodology

Our study was qualitative, framed in the design-based research (DBR) approach (Cobb and Gravemeijer, 2008); it consisted of two design and implementation cycles of the learning trajectory, with third grade children from a public primary school in a low-income (disadvantaged) area of Mexico City. As in the case of Blikstein (2008), we had to design and adapt our proposed learning trajectory activities to the materials to which the disadvantaged children and school had available. The first cycle (2020-2021) took place remotely using Google Meet (due to the COVID-19 pandemic), with a group of up to 16 children (the number of participating students fluctuated due to access issues), over the course of three months. The second cycle (2022) took place face-to-face (in-person), in the first months of the year, with a group of 24 children (when they returned physically, after almost two years of pandemic isolation, to school); this cycle involved 22 sessions, each lasting 1 to 2 hours. The first author (researcher) served as teacher and facilitator, allowing children to engage in active explorations and experiences; the other co-authors acted as observers, taking field notes. All sessions were recorded. We thus collected three types of data: students' written productions, field notes and video recordings. After each session, the data was preliminarily reviewed to determinate any necessary adjustments for subsequent sessions. Later, it was analysed using Davis et al's (2015) classification of spatial reasoning skills and actions; specifically those of *sensating* (*imagining, visualizing, and perspective-taking*), *situating* (*dimensional shifting between 2D and 3D, locating and orientating*), *interpreting* (*comparing, relating 2D instructions and representations*), and *constructing* objects. The analysis followed a deductive approach to systematically interpret the data, by constructing corresponding indicators for each category. We assessed children's spatial reasoning through their "thinking-in-change" (Noss & Hoyles, 2015), as reflected in the actions they undertook while engaging with the learning trajectory activities, paying close attention to how the design and the resources provided fostered or supported that thinking.

4. The learning trajectory with Sample tasks and results

Next, we present some of the tasks and sample results from the implementation of the learning trajectory with the children (from both phases: online and in-person). We attempt to give a brief illustration on how some of the proposed activities could foster the development of spatial reasoning (e.g., situating, interpreting, sensating such as visualizing, and perspective-taking –in particular, the establishment of reference systems and recognition of micro, meso and macro-spaces). Due to space limitations, we only give a general overview of some of the activities and tasks, but the entire learning trajectory description is given in Table 1.

Modules	Main activities and tasks
I. Exploring and understanding the phenomenon	1. Risks in my school (group discussion)
	2. What is an earthquake? (what causes them). Search for information in books and the Internet.
	3. Representing the layers of the Earth: Analysing representations of the Earth's layers and images of landscapes formed by tectonic movements. Discussion on how deep a crack caused by a tectonic movement (a meso/macro space), can be Use of Google Maps and Google Earth to sequentially view some sizes of space: the Earth, the country, the city and the school. Creating representations of the Earth's layers with grains (beans, corn, rice) and pasta
	4. a. Estimating, measuring and iterating in a microspace (in the xy plane: a table) b. Measuring and iterating lengths in a known mesospace (the length and width of the classroom) c. Measuring heights (vertical lengths) – students' heights – in a microspace d. Equivalence of units e. Measuring and iterating lengths in a known mesospace (the playing field) f. Estimating vertical lengths in a macrospace (revisiting the depth of a crack caused by a tectonic movement) *
	5. Creating digital routes in known mesospaces (the home-school environment), using Google Maps *
	6. Building a seismograph. (How are earthquakes measured) Discussion on which instruments measure an earthquake and introduction to the seismograph (with a guest geologist). Discussion on what materials (thread, paper, cardboard, wire, etc.) could be used in the construction of the seismograph; and building it. Simulation and interpretation of seismic movements with each seismograph built
II. A general panorama	7. The strength and stability of buildings Discussion on what a building should have to be less likely to collapse in an earthquake. Construction of resistant multi-cube modules with wooden cubes, Lego blocks, sticks and jelly ◊
	8. What to do in case of an earthquake? Discussing the importance of escape plans
	9. a. Escape maps and routes I (designing an escape map of a microspace structure made with multi-cubes) b. Escape maps and routes II (designing an escape map of the classroom's known mesospace on paper; and with Sweet Home 3D www.sweethome3d.com † c. A blind route: Movement through the classroom and/or school yard (mesospaces) with blindfolds to develop awareness and clear language.
	10. Movements in a virtual microspace in Scratch † / LightBot ◊ (associating the sprite's movements with its space and the movement commands)
III. A local panorama	11. My school's data: School's population statistics.
	12. Identifying safe zones: Identifying meeting points and safe areas to go to in case of an earthquake
	13. Measuring the perimeter of the school (using Google Earth) *
IV. Designing an escape map and plan	14. Drawing a map of the school (using Sweet Home 3D † ◊): Drawing the school's escape map
	15. Sharing each team's maps
	16. Determining the best escape plan

Table 1: Overview of the learning trajectory and main tasks. *Added in second cycle. † Unable to carry out in first cycle. ◊ Adapted for second cycle

We describe some of the initial activities and the construction and work of seismographs, where the traces of the pen represent the intensity of the movements. Then, some sample data attempts to illustrate how students, by engaging in outer constructions and tasks, developed “construction in their heads” (Papert, 1991) of systems of reference, recognized views and measured micro and mesospaces; moreover, how their actions promoted their imagination for comprehending large distances in macrospace.

4.1 Exploring the Earth's crust and the movements of tectonic plates

Seismic movements are produced by the collision between tectonic plates in the Earth's crust (a macrospace), and their measurements with respect to the epicenter and depth are expressed in kilometers. In Activity 2, large distances were approached through the representations of the Earth's layers in order to recognize in which layer earthquakes are generated. Therefore, it was necessary to introduce the unit of measurement “kilometer” from iterations of the meter. A kilometer was exemplified through a distance recognized by the students: e.g., the school's main road (approximately one-kilometer distance). With this notion they interpreted what would be a vertical distance of 35 km from the Earth's crust. In this process of establishing relationships between the different sizes of space, the teacher asked, “If we were in this drawing [the representation of the Earth's layers] and we were standing over here [on the Earth's crust], what would we look like?” Luis commented “very small, very tiny;” Serge said “we would be a microscopic cell” and Samuel thought we would be the size of an ant. Luis, Serge, and Samuel visualized how small a person would look (measurable in meters) if he/she is in a macrospace (kilometers). We feel that the experiences of *measuring*, *interpreting* representations, and *iterating* measurements supported students in recognizing relationships between different sizes in space, units of measurement, and measuring instruments.

4.2 Other spatial reasoning skills developed through the seismograph construction

The seismograph task began with a discussion of predicting / imagining how such a tool would work. Students were then shown a video of a seismograph at work (with the paper tracing the detected movements) so that they could build a homemade seismograph using cardboards, wires, a pen, and paper (Figure 1).



Figure 1: a., b., c., Student-built seismographs.

Through the senses, *perceiving*, *touching* and *seeing*, the students tested and experimented how to adapt the materials they had at their disposal for the construction of a seismograph. For example, they *perceived* how to maintain the balance between the string holding the feather at the top and the string at the back of the box of the seismograph, and thus preventing the feather from falling (Figure 1a). Also, they saw that the string holding the feather had to be thick but manageable to bend; one of the

students had a very thin string and, therefore, the feather would fall out. In addition, they *visualized* that the part where the paper passed through had to have an opening that would allow the flow of both the paper and the pen (Figure 1b).

The students were asked about another way to build the seismograph. In terms of how a seismograph would work, the students considered (*imagined*) the movement of a pendulum as a way to measure the movements of the Earth. Also, students' previous experience with the seismic alert, led them to associate sound with the movements of an earthquake, as explained in particular by Daniel in an online session (Figure 1c).

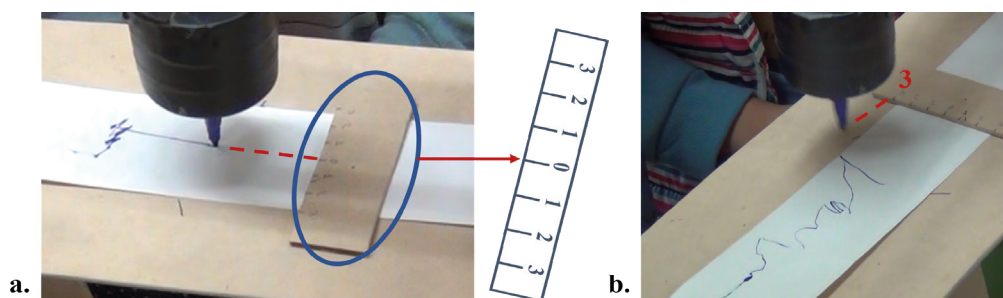


Figure 2: Measuring with a seismograph the intensity of movements (simulated seismic events)

To conclude the activity, the students simulated seismic movements on a seismograph provided by the instructor that recorded the intensity of the movement. The students associated the simulation of the seismic motion with a representation that records that intensity, i.e., they made *dimensional shifting* (between 2D and 3D). They *interpreted* how the bi-dimensional traces on paper, account for the force with which a 3D object was moved. Finally, the students were grouped into groups of five and simulated earthquakes by moving the table. If there was no movement, the pen trace was fairly straight with little range of deviation from the center (Figure 2a). In contrast, if there was motion, depending of its intensity, strokes were recorded with greater deviation from the center (in a numerical range from 1 to 3, on a scale drawn on the side of the strip of paper –see blue-enclosed area in Figure 2a) and without continuity (Figure 2b), with greater deviation for strong motion.

4.3 Transiting between different spaces

In Part 2 of Activity 2 (see Table 1), for children to understand macrospace and estimate measurements of large distances (in particular the depth of Earth's crust layers), they were asked to measure objects in micro and mesospaces. In this way, they had the experience of *visualizing*, *tactilizing*, *imagining (sensating)* and *relating and comparing (interpreting)* measurable qualities of objects in microspaces and distances in mesospaces.

First, in the microspace, children had to find ways of measuring close by objects (e.g., their desks) (Figure 3a) and multicube shapes (Figure 3b). They did this in an *intuitive way* –through *sensating*–, initially by *imagining*, then by using their hands as measuring units (Figure 3a). Through *observing and perspective-taking* of the different views of the 3D multicube objects (see Figure 3b), and *comparing them* with their respective representations in 2D, they could *relate* the two types of representations (3D and 2D, by *identifying* the relationship between shape and representation).

To transit from micro to mesospaces, the following tasks included (i) horizontal distance measurements (measuring one meter on the classroom floor (Figure 3c); measuring 8 meters of the playground), using as tools a measuring ruler, strips of paper (one meter in length) and Google Maps (to calculate the distance from home to school) and (ii) approximation of vertical distances such as the height of the classroom (measuring approximately 3 meters). (These tasks are described in more detail in Ortiz-Rocha, Sacristán & Sandoval, 2024; and Ortiz-Rocha et al., 2022, 2024).



Figure 3: a., b., c., Work in the microspace: a. Measuring a table; b. Recognizing viewpoints (perspective-taking) of a multicube. c. Transiting from micro to mesospaces: constructing and visualizing the length of a meter

In the mesospace, questions such as, "How many times have we used the ruler?" (to measure this distance), "How much do we have so far?" (referring to the measurement), prompted children to estimate and see how many times the measuring ruler would fit in a length (e.g., room height) or distance (e.g., 1 km) to be measured. For the horizontal measurement and *visualizing* 100 cm in length on the floor of the room, most students iterated a 30-cm ruler. For example, Camila used the measuring ruler four times as an unconventional unit, the first three times using it as a whole unit (see Figure 3c) and the last one partially. When the teacher asked: "so far, how much have we got here?" (referring to the measurement), Camila answered: "90, because the first two rulers measure 60 cm and with the last one they would measure 90 in total". These iteration processes are also related to the comparison of distances.

The space to be measured was then enlarged. The children measured the width of the playground with a one-meter strip of paper. Each team had 5 strips and used different strategies (Figure 4a) to iterate this unit approximately 8 times. When comparing the results of these measurements, they reflected on the process of measuring, measurement error, and the correct use of measurement tools.

a. b. c.

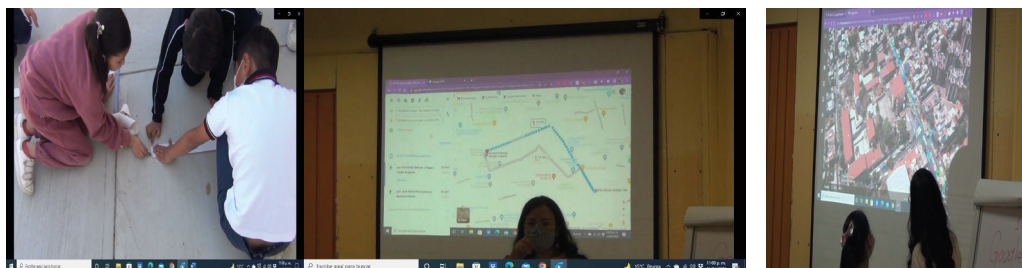


Figure 4: a., b., c., Recognizing mesospaces: a. Measuring the size of the courtyard with paper strips; b. Tracing the commute route in Google Maps; c. Top view of the school

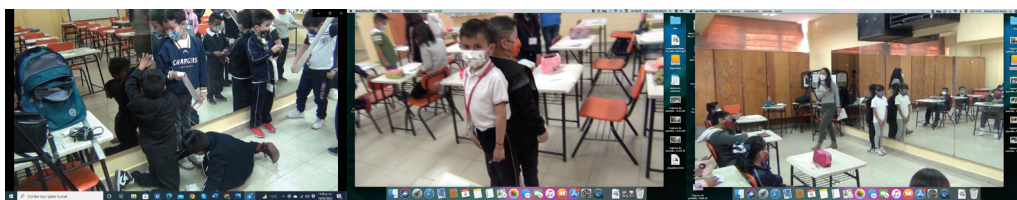


Figure 5: a., b., c., Vertical measurements: a. of students' heights, and b. & c. comparisons for verification

Google Maps was used for students to: (i) see the distance, path, and estimated time to get from home to school (Figure 4b) and (ii) recognize the mesospace of the school (Figure 4c). These tasks not only helped students develop an awareness of distances in macro and mesospaces that would help children imagine the size of Earth's crust but were useful also for later activities where they would be asked to design escape routes in case of an earthquake.

Google Maps allowed children to *visualize* the distance they travel from their home to school and contrast the real commute time, according to the form of transport, with that given by the tool. For example, Leonard contrasted the real commute time of 10 minutes from home to school (950 m), with the 13 min provided by Google Maps (see Figure 4 b); and Ivan travelled 260 meters in 3 minutes (instead of the 5 given by Google Maps). The information provided by Leonard and Ivan made it possible to reflect on the relationship between distance traveled and travel time. For example, the whole class discussed that, although 950 meters seems a long distance, it can be covered in a short time.

For vertical lengths, students: (i) *measured* each other (Figure 5a), and (ii) *estimated* the height of the room. In a first task, they used a strip of paper and the ruler to establish how tall each one was. As a strategy to verify who was taller, they compared their heights directly with each other (Figures 5b and 5c).

For estimating the height of the room (second vertical distance activity), Luis used a non-conventional unit of measurement (the height of a person) as a reference and, from this, estimated the height of the room. Below is an excerpt of Luis' participation:

Teacher: Can you measure the room? How much does it measure?

Luis: The measurement...? [...] the other time in class you said you measured 1.63m.

Teacher: How tall is the room, more or less?

Luis: About 3 meters.

Ivan: About one meter.

Luis: It can't be a meter because the teacher is 1.63m.

Luis took the height of the teacher as a *reference* in order to approximate a distance that was not possible to measure with the tools available. The size of the space to be measured was then expanded with the teacher asking the students about the height (four stories) of the school building, taking into account that one story is about 3m

high. Some children said it would be 12m tall; others 11m. Luis commented: "twelve, adding three, plus three, plus three, plus three, plus three;" Erick corroborated this with another strategy: "because two floors are six, plus another six, it's twelve."

As we saw in the previous examples, Camila, Luis and Erick performed arithmetic operations such as addition (iterating the same quantity) or multiplication. In the online work, for example, Jose had a 17-centimeter ruler and when measuring his height, he commented that he got 5 times his ruler. Instead of adding 5 times 17, or multiplying these two values, his strategy was to use the distributive property for multiplication $(10 \times 5) + (7 \times 5)$. These types of procedures were reviewed and compared in a whole class discussion, showing the students' different ways of obtaining the same answer.

As already mentioned, the *iteration* of units of measurement, in addition to favoring numerical processes, helped students to move between micro and meso-spaces and to *imagine* and *build* strategies to estimate length/distance measurements, as in the case of Luis. These strategies were also a bridge for students to *visualize* and *estimate* distances and lengths in the macro-space of the Earth's crust layers and for the escape routes.

4.4 Moving in a virtual microspace

Another way to encourage the development of reference frames is using digital resources. Some studies indicate that the awareness of the location and orientation of objects in space can be encouraged with manipulative and digital technologies. For example, there is research that reports experiences with programming languages such as Logo (Esparza, 2005) and Scratch (Francis et al., 2016), that highlights the role of those experiences in the development of spatial orientation and representation skills. Logo and Scratch have interfaces that allow the user to draw and move from the perspective of a turtle (Logo) or cat (Scratch), through movement and rotation commands, promoting the development of reference frames. According to Papert (1980), by giving commands to the turtle, children experience syntonik body learning –thus, we claim, attuning embodied (egocentric) with disembodied (decentred) perspective-taking (Tversky & Hard, 2009). Thus, some educational software or digital games can offer opportunities and spatial representations to visualize 3D objects, recognize cross sections, and identify the various perspectives of such objects. However, it is important to recognize when and how to use digital resources to develop spatial skills (Davis et al., 2015).

In our case, we had planned to use Scratch, but it was not possible in either cycle due to lack of computers. We found, in LightBot¹ (Figure 6), a free alternative, which is accessible on phones and tablets. LightBot is a game that uses a sprite, controlled by programming sequences of commands (by dragging them to a command area), to move from one tile to others and reach all the target blue tiles randomly assigned by the software (Figure 6). When the sprite reaches a blue tile, the user uses the light bulb command to indicate when that target has been reached by changing its colour to yellow, and then continue to another blue tile (until all blue tiles have been changed in colour). The game has 8 levels of difficulty, becoming more challenging with each new command introduced, including LightBot (see methodology section), and designed tasks with them, to encourage the construction of allocentric or

1 <https://lightbot.com/> on mobile phones and tablets. <https://www.numuki.com/game/light-bot/> on computer browsers.

decentred reference frames, in addition to other tasks with non-digital resources and experiences in various spaces, in a learning trajectory. In Ortiz-Rocha, Sacristán & Sandoval (2024) we explored in detail the reference frames developed when students engaged with the LightBot environment, and the spatial reasoning actions that gave evidence of those.

By engaging with the elements in the sprite's world, which is external to the students, they need to develop further the non-egocentric spatial relationships (e.g., through syntonic body learning –Papert, 1980). However, the experiences during the physical tasks (those in the courtyard and with concrete multicubes) –where children had to directly see/perceive the location and orientation of objects/subjects in a physical space– also gave them tools to later visualise and establish relationships between elements in other (virtual) contexts, such as LightBot, allowing them to move from concrete to abstract (Bruce et al., 2015). Thus, the learning trajectory's physical and digital tasks were complementary. Additionally, the collaboration between students also helped in the development of non-egocentric reference frames.

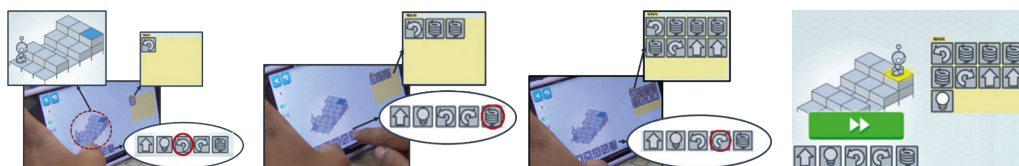


Figure 6: a., b., c., d., A student showing how the sprite is moved in LightBot

5. Discussion and final remarks

Our learning trajectory allowed the participating children to explore and learn about the Earth's movements, the natural phenomenon of earthquakes and what happens when tectonic plates collide, including the formation of volcanoes, mountain ranges and crevasses. It also gave them a context to make sense of daily activities in Mexico City as a highly seismic area. That is, they could engage in authentic inquiry and construction that could help them understand the phenomena while engaging spatial reasoning skills. For example, the participating children were able to recognize/ imagine movements in different space sizes; change between 2D and 3D dimensions, as well as interpret and produce representations in different space sizes in order to design escape routes.

Part of the learning trajectory focuses on the movements of the Earth that are produced in a macrospace and its impact of the meso and microspaces. The construction of a seismograph, in particular, allowed children to see and visualize how to organize and fit together the different provided materials in order to make that technological tool work and be able to register the intensity of movements –a process of engineering systemic thinking. Moreover, the interpretation of the generated representations (traces on paper), as well as the graded scale, made it possible to go deeper into interdisciplinary contents not usually seen in primary school.

Tasks such as the measuring ones –that included the use and iteration of conventional and non-conventional measuring units–, were key for transiting between different space sizes. In microspaces, children *compared* different views of multicube constructions with the physical object (involving changes of dimension) that could be *seen*, *touched*, and *moved*. In mesospaces, they measured and iterated different units. And when measuring vertical distances, for which they didn't have tools available, they did estimations using a known measure. For horizontal distances, in contrast, they

were able to establish relationships between distances and time using Google Maps. We consider that the use of different tools to visualize a known space allowed children to recognize distances of several hundred meters.

In macrospace, children were able to create frames of reference for 2D representations (photos) and visualize large distances by iterating measures in the mesospace. The use and iteration of measuring units in the micro and mesospaces served as a means for *recognizing* and *imagining* distances in macrospace (e.g., the distance from home to school; the depth of Earth's crust layers) through different viewpoints. As mentioned by Galvéz (1985, p. 52), a macrospace is a dimension that can only be conceived through successive local views, separated by movements of the subject on the Earth's surface; we add that this can be done directly or indirectly through the use of digital tools.

One of the main challenges that we faced in our research, was designing a sequence of activities that would be contextualized in a setting that would be familiar to the participating children, with an authentic interaction between the disciplines. Furthermore, its design and implementation had to adapt to the disadvantaged context of the research population where the access and availability of digital resources (mobile technologies, computers, internet) was scarce, whereas the use of recycled materials (for the construction of the seismograph) was easy and fundamental. In spite of the limitations, the results show that it is possible to create integrative STEM constructionist tasks in the context of natural phenomena for developing spatial reasoning skills.

It is worth noting that despite initial challenges when the children returned to school after the pandemic lockdown, they actively engaged and collaborated in the learning trajectory tasks, through which they developed the spatial reasoning skills and reference frames discussed above. A future area of research could be to study, not just the student's spatial actions, but also focus more on the language that they use.

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