Exploring and Developing Geometric Ideas Related to the Concept of Angle Through a Sequence of Microworlds Using Various Digital Tools

Exploring Geometric Ideas Related to the Concept of Angle

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Abstract
This short paper describes a learning trajectory or activities sequence that aims for middle and lower secondary school students to explore and develop geometric ideas and understanding related to the concept of angle, as well as problem-solving skills. The sequence uses various and complementary digital tools (Bridge Constructor, GeoGebra and Logo); each had distinct purpose and constituted a different epistemological domain that allowed students to engage in constructions where angles could be conceived in different ways. The aim of the sequence is for students to develop knowledge of the classification of angles, means to measure them and understand their relationships (e.g. between inner and rotational angles) in different contexts.

Keywords and Phrases: Geometry, Concept of angle, Activity sequence, Bridge Constructor, GeoGebra, Logo.

1. Introduction
An understanding geometric idea related to the concept of angle is fundamental for developing spatial reasoning and analytical skills. However, traditional teaching methods often fail to engage students or connect abstract concepts to practical applications, leading to misconceptions and/or superficial understandings (Giménez, 1990). Thus the need to engage in didactic approaches that favor a gradual and meaningful construction of such concepts, while addressing the epistemic difficulties of the learning of geometry (Giménez, 1990). A constructionist approach (Papert & Harel, 1991), where students can construct and manipulate representations and hypotheses, could foster a deeper understanding and engagement with geometric ideas related to the concept of angle. In a constructionist environment, students take ownership of their learning by building and reflecting on artifacts, whether virtual or physical. This process not only deepens understanding but also encourages creativity and autonomy, as students actively engage with mathematical concepts in meaningful contexts.

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In this paper we present an activity sequence that aimed for middle school and lower secondary students to engage with geometric ideas related to the concept of angle, through a series of small microworlds, some linked to real-world ideas. We aimed, specifically, for students to deepen their knowledge and understanding of the classification of angles (e.g., acute, obtuse, right), how to measure them accurately, and the relationships between angles (e.g., sums of angles around a point in a circle, supplementary relationship between inner angles and rotation angles). Each small microworld was meant to be an environment "where students [could] discover 'nuggets of knowledge' that connect with their prior intuitions and skills" (Noss and Hoyles, 2019, p. 10), and used different digital tools: Bridge Constructor, GeoGebra and Logo.

2. The digital tools in the activity sequence

Each tool used in the sequence had a distinct purpose and constituted a different epistemological domain (Balacheff & Sutherland, 1994):

Bridge Constructor is a game where students build and test bridges. It helps users "simulate real-world structures, exploring geometric principles applied to architectural design" (Clockstone Software, 2011). It provides a realistic context for exploring the application of angles and structural stability, bridging the gap between theoretical geometry and practical engineering. For our purposes, the free (demo) version of Bridge Constructor is sufficient, as it allows students to access all the features necessary to explore the relevant geometric principles within the activity sequence.

GeoGebra (https://www.geogebra.org/) enables dynamic manipulation of geometric figures, allowing students to explore relationships and test hypotheses in real time, and "fostering deeper and more meaningful learning" (Clark-Wilson & Hoyles, 2017, p. 12).

Logo facilitates the construction of geometric patterns through programming, fostering computational thinking and geometric reasoning (Papert, 1980). In our case, we used XlogoOnline (ETH Zurich, n.d.).

Each of these tools provide interactive and practical complementary experiences, fostering engagement so that effective learning can occur, where "students not only construct but also reflect on the public artifacts they create, integrating new ideas into existing cognitive structures" (Ackermann, 2004, p. 7).

3. Methodological considerations

The activity sequence was part of a design-based research (Cobb & Gravemeijer, 2008). In this paper we present the first iteration of the design experiment, which was tested, one-to-one in a clinical setting, with four students, in order to analyze the design of the sequence through the evolution of their geometric reasoning processes and understandings when engaging in the activities. In the next section, we illustrate some of the activities, aiming to provide insights into the pedagogical intentions underlying the design of the sequence, by using data from the case study of a 14-year old student, whom we call Patricia (pseudonym). The collected data included Patricia's digital constructions with each tool, her verbal comments, screen recordings, and field notes. We use her case study to illustrate specific affordances of each microworld and how they contributed to building conceptual understandings of angles in diverse contexts.

4. The activity sequence

As stated above, the activity sequence aimed for students to explore and create meanings for geometric ideas related to the concept of angle: This was conceived as a progressive approach involving three phases: (1) exploration, (2) introduction of new ideas and procedures, and (3) development of the knowledge and meanings created in the previous phases through application in new situations.

4.1 Phase 1

In the exploration phase, the goal is to capture students' attention and motivate them with activities where geometric ideas involving the concept of angle are presented in simulated real-world applications. Activity 1 uses the Bridge Constructor game; its dynamic environment allows students to interact with concepts such as points, line segments, and angles while observing how these elements contribute to the stability of the bridges. By constructing different designs, students not only experience the behavior of angles but also begin to identify and relate these elements to their functionality in real-world structures. Figure 1a presents the construction of a bridge by Patricia. Figure 1b presents another example of a bridge construction.

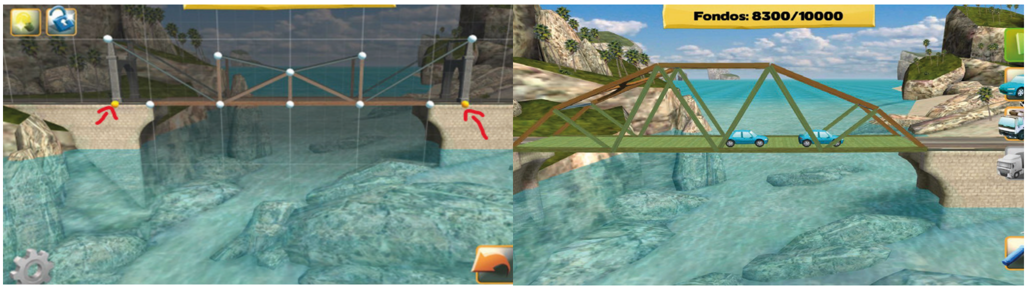

Figure 1: a. Patricia's construction using Bridge Constructor
b: Another student's bridge construction.

In this activity, users have three different types of materials –pillars, wooden beams and cables– to create line segments. Initially, they are only given the yellow points marked with red arrows in Figure 1a, on which they must place the pillars to provide support for the bridge; they then create the horizontal line that represents the bridge using wooden line segments, which measure between 2 and 5 units, and hold these using the cables. By playing with the game, Patricia realized that the best position for the pillars was completely vertical, forming a right angle with the segments representing the bridge. Then, when she attempted to connect the pillar to the bridge, she noticed that the longer the cable –which has no length limit– the wider the angle formed between the cable and the pillar is. Conversely, the closer the cable is placed to the pillar, the smaller this angle gets. Therefore, she decided that the best position is to fix the cable at the point where the first wooden bridge segment ends.

Upon completing the construction, Patricia was able to test, using the game, the bridge's support, simulating the passing of cars and trucks (see Fig. 1b). These experiences helped her understand the relationships between points, line segments, and angles in creating stable constructions. The knowledge developed here established the basis for Activity 2, also part of the exploratory phase.

In this Activity 2, students have to construct a clock with GeoGebra. By interacting with a dynamic model that allows them to observe how angles change as the clock hands move, this activity helps consolidate the previously introduced concepts of angles and their values. Figure 2a shows a clock constructed by Patricia, including the commands used to measure an angle.

Patricia began by generating a circle using the GeoGebra command, placing the center at point A and a different point, named B. Then, she placed a ray starting at A and passing through point B, on the circumference, and completed the clock by adding a third point C on the circumference and generating another ray starting at A and passing through C. Subsequently, she measured the angle <BAC formed by the hands, which, based on the positions of the points in the figure, measured 77.3°. However, thanks to GeoGebra's dragging feature, Patricia had the opportunity to see in real time how the angle's measure changed when moving point C along the circumference. She explored how angles form and change as the clock hands move. This activity reinforced her understanding of angles as measurable geometric entities and introduced the ability to manipulate and observe these concepts in the digital environment. Additionally, this activity serves to develop skills in the use of GeoGebra, which will be crucial to measure the angles in a later activity.

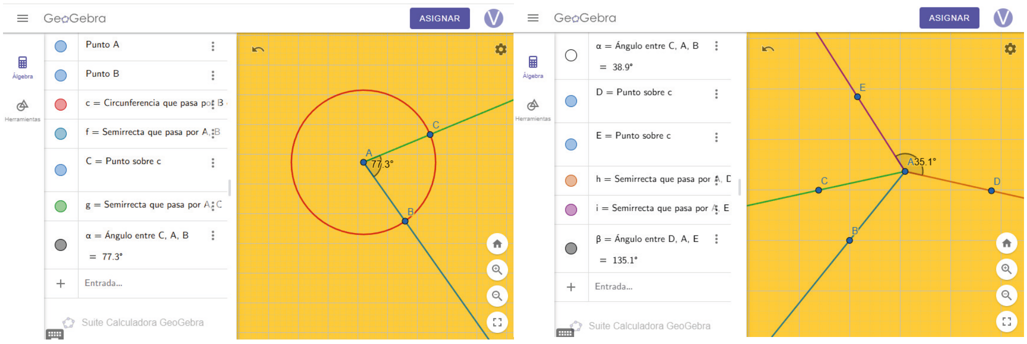

Figure 2: a. Clock created by Patricia in GeoGebra
b. Modified clock construction for Activity 4.

4.2 Phase 2

The second phase aims to introduce new ideas and procedures. It begins with Activity 3, where students' Activity 1 constructions using Bridge Constructor are explored using GeoGebra. In teams, students select a previously constructed bridge and re-construct it in GeoGebra; for this, they need to identify, measure, and classify the angles formed. Figure 3 presents Patricia's GeoGebra re-construction of the bridge that she had built using Bridge Constructor (Fig. 1a).

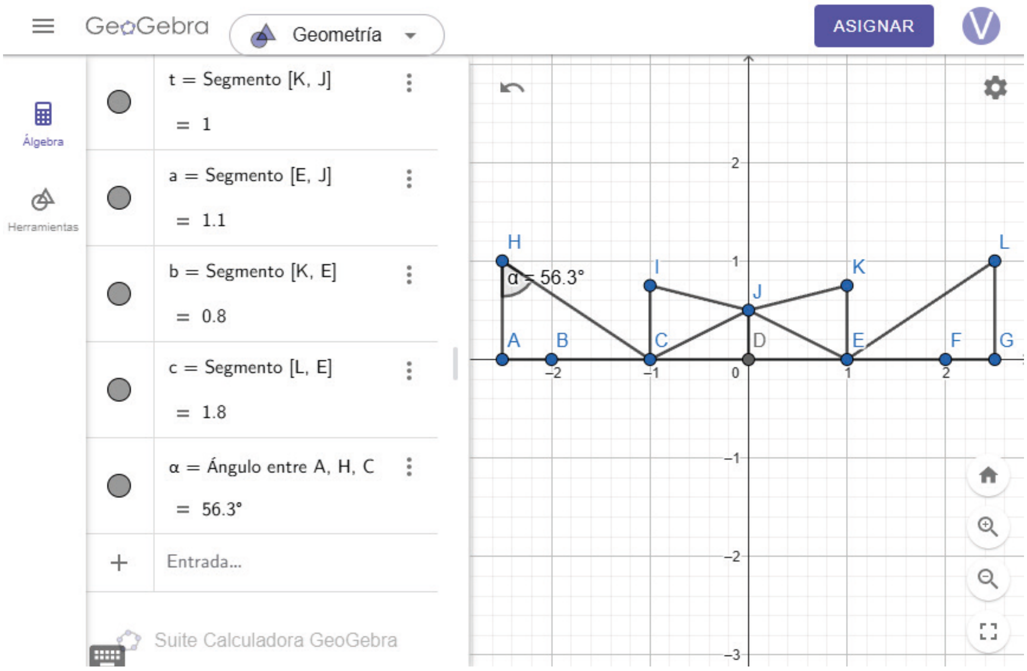

Figure 3: Patricia's GeoGebra representation during Activity 3, of the bridge she had created in Activity 1 (Figure 1a).

This process not only reinforces their understanding of different types of angles but also introduces students to the systematic use of digital tools to analyze and categorize geometric properties. This activity is meant to deepen students' knowledge and understanding of the classification of angles (as acute, obtuse, right) and on how to measure them accurately. Additionally, by constructing, in GeoGebra, a representation of the bridge previously built using Bridge Constructor in Activity 1, students can consolidate their understanding of the relationships between points, line segments, and angles. For example, in Figure 3, Patricia represents the angle α and, with the help of GeoGebra, she can measure it, which she observed to be 56.3°.

Next, in Activity 4, students revisit the clock model constructed in the previous Activity 2, adding new elements, such as line segments (Fig. 2b). This activity consolidates the concepts from Activity 2, requiring again students to identify and classify angles, and leads to the introduction and development of the key concept and property that the sum of angles around a point equals 360°. This emerges when, using GeoGebra, students place the other hands of the clock and obtain the real-time measure of each angle.

Figure 2b shows Patricia's modified clock, where she added line segments. In Activity 4, this student demonstrated a greater mastery of angular concepts. Patricia noticed that, regardless of modifying the positions of the points and thereby changing the numerical values of each angle, the sum of the angles always equals 360°.

This activity not only helps consolidate students' understanding of angle relationships, but it also further develops their ability to manipulate geometric constructions in GeoGebra. Also, the knowledge developed is the basis for Activity 6, where the goal is to introduce the concept of rotational angles through the recreation of a figure in Logo.

The second phase culminates with Activity 5, where, in a group dialogue moderated by the teacher, students share their observations and reflect on the properties of angles, collectively constructing precise definitions that integrate their learning.

4.3 Phase 3

Finally, in Activity 6 in the third phase, students, working individually, use the Logo programming environment to explore rotational angles. Through the programming of geometric figures, students apply the knowledge acquired in the previous phases to analyze and describe how internal angles and rotational angles are related.

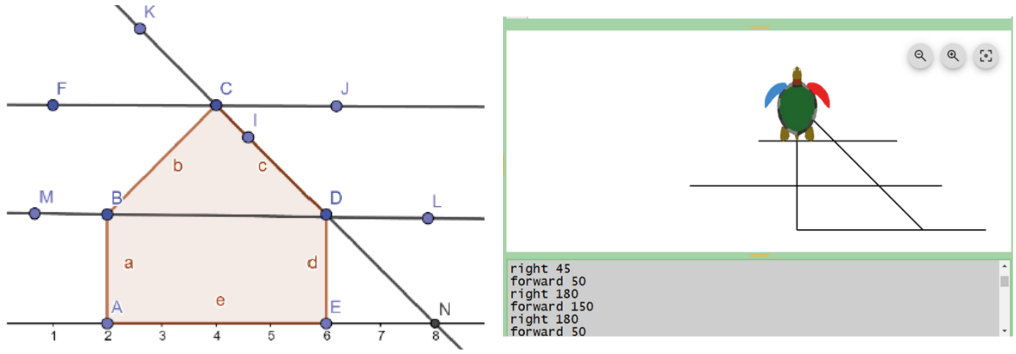

Figure 4: a: Image provided to the students, to be recreated using Logo with the instruction to replicate it using Logo
b: Patricia's attempt to replicate the right side of the given figure

In this last activity, students are expected to replicate a give figure (Fig. 5a) using Logo. By performing the displacements and rotations to generate the new image, using basic commands in Logo such as "forward 50" and "right 180" or "left 180" to rotate an angle and make the turtle move, students should develop ideas related to the concept of rotational angles, as well as relate internal angles to rotational angles. Figure 5b shows Patricia's attempt to recreate the (right side of the) given Figure 5a. Patricia was able to understand rotational angles during her construction of the figure.

Students measured the internal angles of the figures and were able to deduce that the rotation angle was supplementary in order to draw the figure in Logo. This activity reinforces students' understanding of angular relationships, while fostering computational thinking and creativity through programming.

5. Final remarks

This work explored the potential of a sequence of small constructionist microworlds for exploring ideas related to the concept of angle in geometry, aiming to foster deeper understanding, critical thinking, and problem-solving abilities. According to Papert (1980, 1991), the power of constructionism lies in its ability to adapt to diverse learners and provide them with opportunities to build, debug, and refine their knowledge in meaningful ways.

A key result observed during these activities was the increased ability of students to independently construct and justify their understanding of angles in various contexts. More specifically, the activities sequence enabled students to construct, explore, and consolidate geometric ideas related to the concept of angle. We observed, as in the case of Patricia, how they were able to classify angles, measure them and understand their relationship, as was the aim.

However, the work so far has focused on the first iteration of the design of the tasks. Future research will determine better the impact of the sequence on students learning, in order to modify it to be tested in another iteration of the design experiment. It can also explore and include further tasks for this topic, and be tested with more students and diverse contexts, including in authentic classroom settings.

References

Ackermann, E. (2004). *Constructing knowledge and transforming the world.* MIT Media Lab. https://learning.media.mit.edu/publications/Constructing_Knowledge_Ackermann2004.pdf

Balacheff, N., & Sutherland, R. (1994). *Epistemological domain of validity of microworlds: The case of Logo and Cabri-géomètr*e. In R. Lewis & P. Mendelsohn (Eds.), Lessons from Learning: Vol. IFIP Conference TC3WG3.3 (pp. 137–150). Elsevier Science.

Clark-Wilson, A., & Hoyles, C. (2017). Dynamic digital technologies for dynamic mathematics: Implications for teachers' knowledge and practice (Final Report). Nuffield Foundation. https://www.nuffieldfoundation.org/wp-content/uploads/2019/11/Hoyles204190920-20Final20Report.pdf

Cobb, P. & Gravemeijer, K. (2008). Experimenting to support and understand learning processes. In A. E. Kelly, R. A. Lesh & J. Y. Baek (Eds.), *Handbook of design research methods in education* (pp. 68-95). Routledge. https://doi.org/10.4324/9781315759593.ch4

Clockstone Software. (2011). *Bridge Constructor* [Software]. Headup Games. https://play.google.com/store/apps/details?id=com.headupgames.bridgeconstructor&pcampaignid=web_share

ETH Zurich. (n.d.). *XLogoOnline* [Software]. https://xLogo.inf.ethz.ch/release/latest/#/maxi

Giménez, J. (1990). *Obstáculos epistemológicos en el aprendizaje de la geometría.* Revista de Didáctica, 21, 11–21.

Noss, R., & Hoyles, C. (2019). *Microworlds, constructionism and mathematics.* Educación Matemática, 31(2). http://www.revista-educacion-matematica.org.mx/descargas/vol31/2/01_REM31-2_i.pdf

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas.* Basic Books.

Papert, S. & Harel, I. (1991). *Situating constructionism.* In I. Harel & S. Papert (Eds.), Constructionism (pp. 1–18). Ablex Publishing. http://www.papert.org/articles/SituatingConstructionism.html