Designing Manipulatives as Microworld-Making: A Reflective Approach to Enhancing a Course through Pre-Service Teachers' Contributions

Designing Manipulatives as Microworld-Making

Claudia Corriveau[1], and Ivanhoé Moisant[2]

[1] Université Laval, Quebec, Canada
[2] Centre de services scolaire de la Capitale – CSSC, Quebec, Canada,

This paper establishes a foundation for a teacher education course through a reflective approach to its development, viewing it as a dynamic process shaped by students' experiences, knowledge construction, and manipulative design. This perspective reimagines teacher education beyond content delivery, promoting a vision in which both teacher educators and pre-service teachers are simultaneously practitioners, designers, and learners. Consistent with constructionist principles, the study analyses the design work of one student to explore how her contributions informed refinements to course structure and pedagogical framing. The pre-service teacher's manipulative is examined through the lens of Symbolically Structured Environments (SSEs) – conceptually aligned with microworlds – highlighting how physical action interacts with mathematical symbols to deepen understanding. Framing manipulative design as microworld-making, the second iteration of the course incorporated key SSE elements to empower pre-service teachers to design manipulative that actively engage students in mathematical thinking.

Keywords and Phrases: Pre-service Teacher Education, Manipulatives Design, Microworld, Mathematics

1. Introduction

In teacher education, pedagogical resources are typically designed by experts, while teachers and pre-service teachers are seen primarily as users of these materials. This distinction between designers and users reflects an approach that privileges the acquisition and application of knowledge over its creation and contextual adaptation. However, reimagining the teaching profession as one involving the design – not merely the use – of educational resources invites pre-service teachers to adopt more active and reflective roles (Akuom & Greenstein, 2022). Consistent with constructionist principles, this view challenges the conventional separation between learning and design. It sees resource creation not as a technical task but as a knowledge-generating and identity-shaping activity that fosters imagination and continuous adaptations. Guided by this perspective, we developed a course for pre-service mathematics teachers centred on designing manipulative for teaching and learning mathematics. This

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course offered an opportunity to examine how future teachers mobilize empathy and mathematical imagination through the design process (Nicolas-Noir & Corriveau, forthcoming).

The course was conceived not as a fixed sequence of activities, but as an evolving framework continuously enriched by pre-service teachers' experiences and productions. This paper explores how the work of one pre-service teacher in the course's first iteration provided insights that informed a revised version of the course. Such reflection aligns with a model of continuous improvement in teacher education, where pedagogical frameworks evolve iteratively – mirroring the very process of designing manipulatives undertaken by pre-service teachers.

In this paper, we aim to study a pre-service teacher's design project to better ground the design course theoretically. This perspective invites a reimagining of teacher education – not only in terms of content, but also in terms of approach – promoting a vision in which the teacher, whether in training or in a training role, is simultaneously a practitioner, a designer, and a learner.

Naturally, we had already established some theoretical foundations for using design in pre-service teachers' courses. In this context, we first introduce how a course centred on the design of manipulatives can be seen through the lens of constructivism. Next, we outline the structure of the course's initial version and analyse how one pre-service teacher's design work prompted us to refine our pedagogical approach.

2. Drawing FROM Constructionism

Our course invites future teachers to engage in the design of mathematical manipulative, not merely as a technical exercise but as a learning process in its own right. This approach draws on constructionist principles, foregrounding learning through making and reflecting. We align with Dasgupta, Mako Hill, and Monroy-Hernández (2020), who critique the separation often maintained between learning environments and activities, and between designers and learners. By positioning pre-service teachers as both designers and learners, the course deliberately blurs these boundaries. In doing so, it allows them to experience the epistemic value of design – how the act of creating a manipulative can shape and support their understanding of mathematical ideas. Designing a manipulative thus becomes a means for generating knowledge, not only about the mathematics but also about the technology used, and about pedagogical and didactic choices embedded in it.

From this perspective, designing manipulative can be understood as a microworld: a space for immersive, meaningful engagement with mathematical concepts (Noss & Hoyles, 2017). The notion of a microworld has evolved over time. According to Wellner and Levin (2024), it functions simultaneously as a model of reality, a space isolated from reality, and a reflection of reality. Interpreting Papert's analysis of gears as "objects-to-think-with" (Papert, 1980, p. viii), Wellner and Levin use post-phenomenological theory to map Papert's four qualities of microworlds (in parentheses) onto different relational categories : background relations (gears as part of a natural landscape); alterity relations (the learner's engagement with gears as interactive objects); embodiment relations (the bodily experience involved in using gears); and hermeneutic relations (gears as interpretive tools for understanding concepts). The activity of designing a manipulative as a pre-service mathematics teacher closely aligns with Wellner and Levin's lens, this design activity can be seen as engaging

the future teacher in different relational modes: as they imagine the manipulative within a classroom setting (background), interact with evolving prototypes (alterity), engage bodily in the making and testing process (embodiment), and reflect on the educational meaning of their creation (hermeneutics). In this way, the design process itself becomes a microworld – a space where thinking, making, and learning are deeply intertwined.

3. The course setting cycle 1 and the designed manipulative

Not all the designed objects had the same mathematical value. During the first iteration of the course, which took place in 2023, five final-year pre-service teachers participated as part of their four-year teacher education program. Of these five, two created manipulative intended for a demonstration by the teacher. The other three designed manipulative meant to be used directly by students. One of these pre-service teachers' projects (Camille's) led us to reflect more deeply on the role of manipulative and how the course could be restructured to better support such design processes. The work she carried out was intuitively compelling, both in the process she engaged in and the material she produced. Our analysis took a somewhat reversed approach: rather than starting with a theoretical framework to guide the design, we began by closely examining the process and the mathematical potential of the manipulative created. We then turned to the literature to identify the concepts and perspectives that could best account for what had taken place. We therefore wanted to provide a theoretical grounding for both the process and the resulting object. To this end, we drew on the work of Hendrick, as well as that of Coles and Sinclair.

Students had full autonomy over their chosen topics, process and manipulative designs. They were supported by the course teacher (first author) and an educational technologist with expertise in designing tangible learning tools (second author). Students learned to use Tinkercad© (2016) and Onshape© (2020) through tutorials and had access to 3D printers. They had to write a Design Journal to document key elements of the process: evolving ideas, challenges encountered and intended classroom uses of the manipulative. Camille's design process followed Henriksen et al. (2020) design thinking model for teacher education, including five phases: "empathize, define, ideate, prototype, and test". She designed a manipulative centred on integers, linking physical actions with mathematical rules. Her design emphasized *embodied relation*, encouraging students to coordinate physical manipulation, bodily movement, and symbolic representations. Table 1 present a brief description of the manipulative introduced through the five phases of its development.

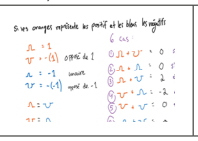
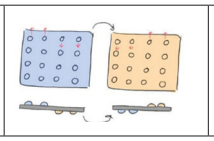
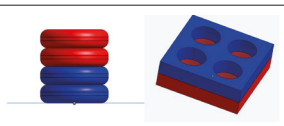
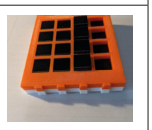
Empathy	She clarified the reasons for choosing integers, writing (Journal p. 4): *I decided to explore this concept further since operations with negative numbers are a challenge for students, according to my observations, even in secondary school.*
Defining	She defined what would be covered by the manipulative (Journal, p. 4): *My experiences during internships and tutoring have revealed that students often confuse the different meanings of the "-" (the sign of the number, the operator, the opposite). Therefore, I hope to design a manipulative that will help clarify the various meanings of "-".*
Ideating	She explored different existing models of integers (tokens, rational line, etc.). She draws ideas from the limit of these models:
Prototyping	   
Testing	She designed three tasks. Each task building on the previous: 1) students act on a number and its opposite, compare actions, and infer equivalence; 2) students must achieve a result using a fixed number of pops, introducing variation in operations; and 3) one student pops to represent an operation; another observes the opposite side and compares results.

Table 1: Description of the manipulative introduced through Henriksen et al. five phases

To analyse the novelty of this design, we drew on the concept of Symbolically Structured Environments (SSEs), as developed by Coles and Sinclair (2019a, 2019b). SSEs organize physical actions within an environment structured by mathematical relationships and offer opportunities for symbol use from the outset. These environments constrain and support action, enabling the emergence of mathematical fluency. According to Coles and Sinclair (2019a), SSEs are akin to Papert's microworlds: immersive environments where students transfer personal habits of exploration into formal domains. SSEs aim to promote fluency while supporting mathematical structuring and exploration. We used the five characteristics of SSEs – as described in table 1 – to analyse Camille's manipulative.

She named her manipulative the Numeric Pop-it (Figure 1). It aimed to help students distinguish between the binary and symmetric meanings of the minus sign through embodied interaction. Pressing a button ("popping-in") represented subtracting one unit; pulling a button ("popping-out") represented adding a unit. The two sides of the Pop-it were connected, so popping in on one side corresponded to popping out on the other – thus linking the symmetric notion of integers with physical reversibility. This design aligns with SSE principles. We also noted that it is as well closely related to Wellner and Levin's relational framework, both referring to the notion of microworld (see table 2).

Principles of SSE	Pop-it actions	Relational framework
a) Symbols stand for actions	The minus sign represents both the action (popping) and the reference colour	• Background relation: Mathematical ideas emerge naturally through play • Alterity relation: The Pop-it becomes an interactive partner in learning • Embodiment relation: Physical manipulation aids in understanding abstract rules • Hermeneutic relation: The Pop-it mediates conceptual understanding of integers
b) Symbol use is governed by embedded mathematical constraints	Popping actions and colour reversals are consistent with subtraction/addition rules	
c) Operations are linked to their inverses	Both popping and flipping are reversible operations	
d) Complexity is constrained but situated within a mathematically coherent whole	Tasks were structured to increase in complexity and enable new symbolic combinations. These tasks provided entry points for exploring inverse operations, symbolic equivalence, and strategy variation – illustrating how students could make novel symbolic moves.	
e) Novel symbolic moves are possible		

Table 2: Principles of SSE, Pop-it actions and Relational Framework

This backward-looking analysis allowed us to make sense of the work in a deeper way and, in turn, informed a revision of the course. By grounding our observations in theory after the fact, we were able to refine the course design to better support future iterations and strengthen its educational foundations.

4. Revisiting the Course: Designing as Microworld-Making

For the second iteration of the course, we reframed manipulative design as microworld-making, explicitly drawing on the Symbolically Structured Environments (SSE) framework. This shift encouraged pre-service teachers to create learning environments that foster student agency and exploration, where knowledge is constructed through interaction with the manipulative.

To support this reframing, we partnered with a design school to explore how professional designers structure their creative processes. This collaboration led us to introduce a comprehensive design brief, which included key elements such as target audience, educational purpose, material function, constraints and affordances, expected deliverables, timelines, stages, and budget.

Unlike in the first version, we organized group and subgroup activities to support the design process and clarify expectations. For example:

- We collaboratively developed a definition of manipulative with the pre-service teachers, drawing on Bartolini & Martignone (2020): artefacts handled by students to learn mathematics.
- We facilitated classroom discussions on the role of the body in learning mathematics.
- We held a hands-on workshop in which pre-service teachers had access to various crafting tools (paper, sticks, glue, scissors, etc.) to help them prototype and test a paper manipulative involving movement and designed around the concept of integers.

- We encouraged some pre-service teachers to start from common student difficulties and to compare different models of the chosen mathematical concept in order to generate design ideas.
- As part of the design constraints, we required students to draw on SSE principles to guide how physical actions could be meaningfully structured by mathematical relationships. We provided Camille's Pop-it as an example of such a manipulative.

In 2025, eleven pre-service teachers participated in the second iteration of the course (January to April), engaging in design projects informed by this microworld-making approach. Each designed a manipulative or game intended to be handled by students, covering a range of mathematical concepts such as probability, relationships between quantities, fractions, the Cartesian plane, angles, and percentages.

Conclusion

In this paper, we have presented how the second iteration of a course for pre-service teachers was redesigned around the idea of manipulative design as microworld-making. Drawing from the SSE framework and inspired by professional design practices, we structured the course around a comprehensive design brief and collaborative activities that guided students in creating manipulative grounded in mathematical meaning. This evolution emerged from close observation of pre-service teacher's work during the first version of the course and a desire to deepen the theoretical underpinnings of her (our) approach.

Interestingly, the course itself and the manipulative designed within it mirror one another in both structure and intention. Just as the manipulative were conceived as microworlds – environments rich in possibilities for exploration and meaning-making – the course itself became a microworld for the pre-service teachers. Through collaborative design, they engaged in a relational process of construction and reflection. This "mise en abyme" between form and content reveals a strong coherence between our course structure and the theoretical lenses we mobilize, particularly in terms of learners' agency.

Finally, this work also invites us to consider how we learn from students' experiences. Rather than focusing solely on their misunderstandings to improve our teaching, we choose to draw inspiration from their creative ideas – those that spark our curiosity and make us want to articulate them within research frameworks.

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